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A SIMPLE ADVERTIVE MODEL

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¶ This is an attempt to devise a two-parameter model that has the following characteristics:

1) Preserves the desirable features of the most familiar models that have been tested extensively in the past — namely, the barotropic and two-level baroclinic models.

2) Does not include the previously undesirable features of these models.

3) Is so simple in its mathematical expression that one can readily see just how the forecast evolves from a given initial state.

¶ With regard to the first of these points, it is to be noted that the 500 mb barotropic forecasts made from balanced initial conditions are quite good up to 24-48 hours — better than the 500 mb thermotropic forecasts. The latter, when made from initially geostrophic wind fields, suffer from spurious anticyclogenesis along the west coast and off the Atlantic seaboard. Even when made from initially balanced wind fields, the 500 mb thermotropic forecasts sometimes show a tendency toward excessive cyclogenesis, particularly when the thickness gradient is very strong.

P There is, in my opinion, serious question that the intensification of troughs at 500 mb is due, in physical reality, to the so-called "development" term. More obviously, it is due to convergence, which depends on the character of the ω-profile and, in turn, on the thermodynamic processes. But the coefficient of the "development" term is independent of the static stability. Hence, that term probably owes its existence mainly (and entirely) to the fact that the sum of products is not the product of sums — i.e., to the particular type of finite-difference scheme one applies to a nonlinear equation. It appears probable, therefore, that it is as valid to apply the nondivergent barotropic equation at 500 or 600 mb (roughly midway between surfaces where ω is small) as it is to apply the thermotropic equation at 500 mb or any other surface.

P Accordingly, one of the fundamental equations for the simple model under consideration is the nondivergent barotropic vorticity equation, applied (for instance) at 600 mb. The initial conditions shall be the 600 mb streamfunction. The motions at 600 mb will then be completely independent of those at any other level, and will be highly "conservative".

¶ The height gradients at 1000 mb are generally due to rather small differences between the gradients of 500 mb height and gradients of 1000-500 mb thickness, and are accordingly very sensitive to errors in either of the latter. Since the use of balanced initial conditions generally removes large errors in 500 mb height gradient over the U.S. and N. Atlantic, the main concern is to remove errors in the predicted thickness pattern. At the present time, the barotropic forecasts of thickness suffer from:

1) A tendency toward excessive amplitude, and
2) A tendency to move the thickness pattern along as if it were unconnected with the 500 mb flow, and at a rather erratically different speed.

¶ As a result, the 1000 and 500 mb height patterns often bear an unrealistic phase relationship to each other. This, coupled with overamplification of thickness fluctuations, frequently results in explosive developments in the 1000 mb height pattern. On the whole, the thickness is not conservative enough — taken either literally or figuratively. One question, therefore, is the following: Are the thickness fields approximately conservative, in the sense that its changes are due to the "advection" of thickness in a horizontal vector field? The latter will not, of course, coincide exactly with the wind field, but may be as smooth and regular in its behavior. If so, the mathematical expression of this "conservation" might provide a means for subduing the erratic behavior now shown by the thermotropic forecasts.

¶ Moreover, the behavior of such an advective or conservation model could be much more readily understood than one in which thickness changes are reflected in several different types of mathematical operations.

¶ We begin by deriving an equation for ω in the usual fashion, subtracting the vorticity equation at 800 mb from that at 400 mb, introducing finite-differences, and replacing the true vorticity by geostrophic vorticity

$$\frac{\partial}{\partial t}\nabla^2 h + J(\psi, \nabla^2 h) + J(h, \nabla^2 \psi) + J(h, f) = \frac{2f\eta\omega}{gP} \quad (1)$$

where h is the thickness between 800 and 400 mb, ψ is the streamfunction at 600 mb, η is the absolute vorticity at 600 mb, and P is 400 mb. Everything else is standard. The adiabatic equation at 600 mb is:

$$\frac{\partial h}{\partial t} + J(\psi, h) = \frac{\sigma^2 \omega}{gP} \equiv W \qquad (2)$$

in which $\sigma^2 = \frac{R^2 T^2}{g\theta}\frac{\partial \theta}{\partial z}$. Eliminating $\frac{\partial h}{\partial t}$ between (1) and (2),

$$\mu^2 W - \nabla^2 W = J(\psi, \nabla^2 h) - \nabla^2 J(\psi, h) + J(h, \nabla^2 \psi) + J(h, f) \quad (3)$$

where $\mu^2 = \frac{2f\eta}{\sigma^2}$. We now note that

$$\nabla^2 J(\psi, h) = J(\psi, \nabla^2 h) - J(h, \nabla^2 \psi)$$

$$+ 2\left(\frac{\partial}{\partial x}\nabla\psi \cdot \frac{\partial}{\partial y}\nabla h - \frac{\partial}{\partial y}\nabla\psi \cdot \frac{\partial}{\partial x}\nabla h \right)$$

whence Eq. (3) becomes:

$$\mu^2 W - \nabla^2 W = 2J(h, \nabla^2\psi) + J(h, f) - 2\left(\frac{\partial}{\partial x}\nabla\psi \cdot \frac{\partial}{\partial y}\nabla h - \frac{\partial}{\partial y}\nabla\psi \cdot \frac{\partial}{\partial x}\nabla h\right)$$

The important point is this: In normal conditions, when the flow is almost equivalent barotropic, the third term on the right hand side of this equation is small, whereas the others are not. Approximately, then,

$$\mu^2 W - \nabla^2 W = J(h, 2\nabla^2\psi + f)$$

Moreover, because:

(a) W oscillates around zero (and is highly correlated with its Laplacian), and

(b) the left hand side of this equation is the sum of terms of the same sign,

$$W(\mu^2 + \alpha^2) \simeq J(h, 2\nabla^2\psi + f) \; .$$

where α is the characteristic wavenumber of the W-field. Thus, regarding μ and α as slowly-varying functions

$$\boxed{\frac{\partial h}{\partial t} + J\left(\psi + \frac{2\nabla^2\psi + f}{\mu^2+\alpha^2}, h\right) = 0} \qquad (4)$$

According to this formula, the thickness pattern is advected along isolines of $\left(\psi + \frac{2\nabla^2\psi + f}{\mu^2+\alpha^2}\right)$, and at a speed that is proportional to its gradient.

¶ We now investigate the general behavior of the thickness field predicted by this formula. We first let

$$\psi = \Psi + \psi'$$

where Ψ is a function of latitude only (very nearly linear), such that $\nabla^2\Psi = 0$. Thus

$$\nabla^2\psi = \nabla^2\psi'$$

Moreover ψ' oscillates around zero, so that

$$\nabla^2\psi' \simeq -\alpha^2\psi'$$

Approximately, then, Eq. (4) is:

$$\frac{\partial h}{\partial t} + J\left(\Phi + \psi' - \frac{2\alpha^2\psi'}{\mu^2+\alpha^2} + \frac{f}{\mu^2+\alpha^2}, h\right) = 0$$

Now, according to the linear theory of baroclinic instability, the wavenumber of "first" instability is given by

$$\alpha^2 = \frac{\mu^2}{\sqrt{2}}$$

Thus,

$$\frac{2\alpha^2}{\alpha^2+\mu^2} \simeq \frac{\sqrt{2}}{1+\frac{1}{\sqrt{2}}} = \frac{1.414}{1.707} \simeq .8$$

and

$$\frac{\partial h}{\partial t} + J\left(\Phi + .2\psi' + \frac{f}{\mu^2+\alpha^2}, h\right) = 0$$

Very nearly, then,

$$\frac{\partial h}{\partial t} + \left(U - \frac{\beta}{\mu^2 + \alpha^2} \right) \frac{\partial h}{\partial x} = 0$$

This states that the thickness pattern is advected toward the east at the speed

$$c_h = U - \frac{\beta}{\mu^2 + \alpha^2} \qquad (5)$$

where U is an average wind speed toward the east. Under more general conditions, it is easy to see that the thickness pattern is advected along a vector that is more nearly oriented east-west than the wind. This could not generally lead to amplification of the thickness fluctuations.

¶ For the middle latitudes, and for reasonable values of μ and α

$$\frac{\beta}{\mu^2 + \alpha^2} \simeq 300 \text{ miles / day.}$$

Thus, the thickness pattern predicted by Eq. (4) would move at about 2/3 the speed of the average westerlies, a conclusion that is adequately supported by observation.

P Finally, it should be pointed out that the speed of the streamline pattern at 600 mb is about

$$c_\psi = U - \frac{\beta}{\alpha^2}$$

Thus, the speeds of the 600 mb streamline pattern and the thickness pattern would be about the same, whence <u>the initial phase relationship would be preserved or changed slightly in the direction of occlusion</u>.

P <u>Proposed computing procedure</u>:

1) Interpolate between 400 and 850 mb to get initial 600 mb height.
2) Solve balance equation for initial ψ at 600 mb.
3) Compute initial h- field from heights at 400 and 850 mb. Note! <u>This would avoid spurious thickness</u>

gradients due to height-reduction to sea level or 1000 mb.

4) Predict ψ by inverting barotropic vorticity equation.
5) Predict h from Eq. (4), using a smoothed values of $\nabla^2\psi$, and choosing a suitable constant for (μ^2+a^2).

For printout: Invert balance equation to get 600 mb height from 600 mb streamfunction. Subtract half predicted h to get 800 mb height, and extrapolate to get height at lower levels.